

**Hale School**

**MATHEMATICS**

**SPECIALIST**

**3CD**

**Semester Two Examination 2011**

**MARKING KEY and SOLUTIONS**

**Section One**

**Calculator-Free**

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**Question 1 [9 marks]**

Give exact values for the following :

(a) 

[1]

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| **Solution** |
| = cis(-π/2) = - i |
| **Specific Behaviours** |
| ✓ Correct answer |

(b) 

[2]

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| **Solution** |
| =  = cis(-π) = -1 |
| **Specific Behaviours** |
| ✓ Recognises cis with the correct argument  ✓ Uses DeMoivre’s Theorem to multiply argument correctly and give the correct answer |

(c) 

[3]

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| **Solution** |
| =  =  = |
| **Specific Behaviours** |
| ✓ Recognises the limit as a derivative  ✓ Correct identification of the function cos 2x  ✓ Determines the exact value |

(d) The shaded area under the curve y = cos 2x.

[3]

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| **Solution** |
| square units |
| **Specific Behaviours** |
| ✓ Correct expression for the area  ✓ Anti-differentiates correctly  ✓ Correct evaluation |

**Question 2 [9 marks]**

Given that z = eix and w = e–ix (where x is a real number) :

(a) express cis(3x) w in terms of z.

[2]

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| **Solution** |
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| **Specific Behaviours** |
| ✓ Expresses cis(3x) in terms of a complex exponential  ✓ Correct expression in terms of z |

(b) if z - w is expressed in the form a + bi determine the values of a and b.

[2]

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| **Solution** |
| Hence a = 0, b = 2 sin x |
| **Specific Behaviours** |
| ✓ Recognises expression to give twice the imaginary part  ✓ Correct values for both a and b. |

(c) simplify z3 + w3

[2]

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| **Solution** |
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| **Specific Behaviours** |
| ✓ Correct use of index laws with the complex exponential  ✓ Simplifies correctly in terms of twice the real part |

(d) solve for x given that z4 + 1 = 0

[3]

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| **Solution** |
| Hence x =  ,  ,  , |
| **Specific Behaviours** |
| ✓ Expresses -1 in polar form with argument π  ✓ Correct expression for the 4 solutions in polar form  ✓ Correct values for x (using convention between -π to π) |

**Question 3 [6 marks]**

Points A, B and C have respective position vectors given by :

**a** = **i** + **j**  - **k**

**b** = **i** + **j**  + **k**

**c** = 2**i** + **j**

Determine :

(a) the value of cosine of the angle between vectors **a** and **b**.

[2]

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| **Solution** |
| **a . b**  1 = 3 cos θ  Hence cos θ = |
| **Specific Behaviours** |
| ✓ Correct use of dot product and the magnitudes of each vector  ✓ Correct value for cos θ |

(b) the vector equation of the line containing points A and B.

[2]

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| **Solution** |
| Direction vector **d = b - a** =  Vector equation line **r** = |
| **Specific Behaviours** |
| ✓ Finds an appropriate direction vector  ✓ Expresses in correct point-direction vector form (does not have to express as a single vector) |

(c) the vector equation of the plane containing vectors **a** and **b** and also containing the point C.

[2]

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| **Solution** |
| Vector equation plane **r** =  Alternative answer **r .** |
| **Specific Behaviours** |
| ✓ Uses 2 parameters with the direction vectors a and b  ✓ Expresses in correct vector form (does not have to express as a single vector)  NO MARKS if students uses vectors a and b as points in the plane |

**Question 4 [3 marks]**

Evaluate the definite integral  exactly :

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| **Solution** |
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| **Specific Behaviours** |
| ✓ Correct use of the cosine DOUBLE angle identity  ✓ Anti-differentiates correctly  ✓ Correct evaluation  NO marks for use of sin3x as the anti-derivative |

**Question 5 [4 marks]**

(a) Determine matrix T = 

[1]

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| **Solution** |
| T = |
| **Specific Behaviours** |
| ✓ ALL Matrix elements are correct |

(b) Hence if matrix T represents a transformation matrix, describe the actions of matrix T.

[3]

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| **Solution** |
| T = A B i.e. B then A  i. Reflect about the line y = -x and then  ii. Dilate horizontally about x = 0 with factor 2 |
| **Specific Behaviours** |
| ✓ Description of the reflection matrix  ✓ Description of the dilation matrix  ✓ Correct order i.e. reflect then dilate |

**Question 6 [5 marks]**

The natural logarithm function can be defined as ln(x) =  where x > 0.

(a) Given that a, b > 0, using the substitution u =  find an expression for the definite integral  .

[3]

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| **Solution** |
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| **Specific Behaviours** |
| ✓ Changes limits correctly  ✓ Expresses integrand correctly  ✓ Recognises answer using the natural logarithm definition |

(b) By considering  =  +  and using the result from part (a) make a deduction about the natural logarithm function.

[2]

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| **Solution** |
| =  +  ln (ab) = ln a + ln b |
| **Specific Behaviours** |
| ✓ Uses the result from part (a)  ✓ Deduces that the log(Product) = sum of logarithms |

**Question 7 [4 marks]**

Prove, by any method, that the cube of any number that is 2 more than a multiple of 3 is always 1 less than a multiple of 9.

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| **Solution** |
| Let n be any counting number.  Hence 3n + 2 is two more than a multiple of 3 (the particular number)  Consider (3n + 2)3 = (3n)3 + 3(3n)2(2) + 3(3n)(22) + 23  = 27n3 + 54n2 + 36n + 8  = 27n3 + 54n2 + 36n + 9 - 1  = 9(3n3 + 6n2 + 4n + 1) - 1  Hence (3n + 2)3 is always of the form 9k - 1 |
| **Specific Behaviours** |
| ✓ Express the cube of the particular number  ✓ Expand correctly the cube of the binomial  ✓ Simplify each term correctly  ✓ Express in the form 9k - 1 |